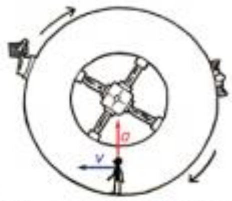
NAME ______________________ DATE ____________

Scenario

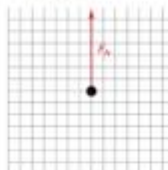
A doughnut-shaped space station is built far away from the gravitational fields of Earth and other massive bodies. For the comfort and safety of the astronauts, the space station is rotated to create an artificial internal gravity. The rotation speed is such that the apparent acceleration due to gravity at the outer surface is 9.8 m/s². The space station rotates clockwise.



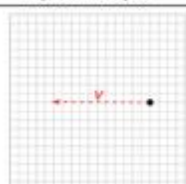
Using Representations

PART A: On the image at right, sketch and label vectors that represent the astronaut's velocity and acceleration.

PART B: The dot at right represents the astronaut standing in the space station. Draw a free-body diagram showing and labeling the forces (not components) exerted on the astronaut at the instant shown. Draw the relative lengths of all vectors to reflect the magnitudes of all the forces.



PART C: The astronaut drops a ball. On the following diagrams, sketch the velocity and acceleration vectors for the ball as seen by an observer outside the space station in an inertial frame of reference. These are NOT free-body diagrams.

After the ball is released and before it hits the floor	*After the ball hits the floor*
	

PART D: From the point of view of a person watching from outside the space station, what does the path of the ball look like?

A person outside the space station sees the ball travel directly to the left until it impacts again with the "floor."

PART E: From the point of view of the astronaut inside the space station, what does the path of the ball look like?

The astronaut inside the space station sees the ball appear to be pulled "downward" and to the right (from the astronaut's point of view).

NAME ______________________ DATE ____________

Scenario

The mass of Mars is 1/10 times that of Earth; the diameter of Mars is 1/2 that of Earth.

Quantitative Analysis

PART A: Derive the equation for gravitational, g, due to a planet.

$$F_g = \frac{GM_1M_2}{R^2} = M_2 g$$

$$g = \frac{GM_1}{R^2}$$

PART B: Let g be the gravitational field strength on Earth's surface. Derive an expression for the gravitational field on the surface of Mars without plugging in a value for the mass or radius of Mars. Your answer should be a number multiplied by g. For each line of the derivation, explain what was done mathematically (i.e., annotate your derivation).

$g_{Earth} = \frac{GM_{Earth}}{(R_{Earth})^2}$	The acceleration due to gravity on Earth's surface is related to the mass and radius of Earth by:
$g_{Mars} = \frac{G\left(\frac{1}{10}M_{Earth}\right)}{\left(\frac{1}{2}R_{Earth}\right)^2}$	Plugging in the relationships for the mass and radius of Mars in terms of the mass and radius of Earth:
$g_{Mars} = \frac{\frac{1}{10}}{\frac{1}{4}}\frac{G(M_{Earth})}{(R_{Earth})^2}$	Simplifying
$g_{Mars} = \frac{4}{10}\frac{GM_{Earth}}{(R_{Earth})^2}$	Simplifying
$g_{Mars} = \frac{4}{10}g_{Earth}$	The acceleration due to gravity on Mars is approximately $\frac{4}{10}$ the acceleration due to gravity on Earth, or about 4 m/s^2.

Argumentation

PART C: A rock is dropped 2.0 meters above the surface of Mars. Does this rock take a longer or a shorter time to fall than a rock dropped 2.0 m above the surface of Earth? Justify your answer without using equations.

The acceleration due to gravity on Mars is less than on Earth. Therefore, it will take longer to fall the same distance on Mars.

PART D: On the internet, a student finds the following equation for the time an object will take to fall to the ground from a height h, depending on the mass and radius of the planet the object is on: $t = \sqrt{\frac{2hG}{MR^2}}$

Regardless of whether this equation is correct, does it agree with your qualitative reasoning in Part C? In other words, does this equation for t have the expected dependence as reasoned in Part C?

__X__ Yes ______ No

Briefly explain your reasoning without deriving an equation for t.

Both the mass and the radius of Mars are smaller than the mass and radius of Earth, which leads to a smaller acceleration and a larger time for a constant height. This equation shows that if M and R are both smaller, the time to fall should be larger, which is what we predicted.

PART E: Another student deriving an equation for the time it takes for an object to fall from height h makes a mistake and comes up with: $t = \sqrt{\frac{R^2}{2GMh}}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

If you increase the height that the object is falling from, the time shouldn't decrease, which is what this equation says. Also, the units do not give seconds if analyzed.

NAME ______________________ DATE ____________

Scenario

A student is given the following set of orbital data for some of Jupiter's moons and is asked to use the data to determine the mass M_J of Jupiter. Assume that the orbits of these moons are circular.

Orbital Period T (seconds)	*Orbital Radius R* (meters)	(s^2)	(m^3)
2.08×10^{7}	1.12×10^{10}	4.3×10^{14}	1.40×10^{30}
2.49×10^{7}	1.26×10^{10}	6.2×10^{14}	2.00×10^{30}
4.05×10^{7}	1.71×10^{10}	16.4×10^{14}	5.00×10^{30}
5.03×10^{7}	2.02×10^{10}	25.3×10^{14}	8.24×10^{30}

Create an Equation

PART A: Write an algebraic expression for the gravitational force between Jupiter and one of its moons.

$$F_g = \frac{GM_JM_m}{R^2}$$

PART B: Use your expression from Part A and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

$F_g = \frac{GM_JM_m}{R^2}$	The force of gravity between Jupiter and one of its moons can be written
$\frac{GM_JM_m}{R^2} = \frac{M_mv^2}{R}$	Where the force of gravity is the centripetal force, responsible for keeping the moon in orbit
$\frac{GM_J}{R} = v^2$	Canceling R and the mass of the moon
$\frac{GM_J}{R} = \frac{4\pi^2R^2}{T^2}$	Substituting in $\frac{2\pi R}{T}$ for the velocity
$M_J = \frac{4\pi^2R^3}{GT^2}$	Solving for the mass of Jupiter
$T^2 = \frac{4\pi^2R^3}{GM_J}$	Period squared as a function of radius

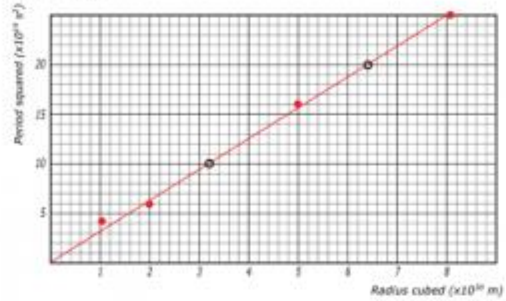
Data Analysis

PART C: Which quantities should be graphed to yield a straight line whose slope could be used to determine the mass of Jupiter?

The period squared should be graphed vs. the radius of orbit cubed.

PART D: Complete the table by calculating the two quantities to be graphed. Label the top of each column, including units.

PART E: Plot the graph on the axes below. Label the axis with the variables used and appropriate numbers to indicate the scale.



PART F: Two identical probes are sent to study one of Jupiter's moons. Probe A is in geosynchronous orbit around the moon while probe B rests on the surface of the moon and rotates with the moon.

Rank the magnitudes of the following gravitational forces from greatest to least. If two or more quantities are the same, say so clearly.

a. The force of the moon on probe A

b. The force of the moon on probe B

c. The force of probe A on the moon

d. The force of probe B on the moon

e. The force of probe A on probe B

f. The force of probe B on probe A

Greatest B = D > A = C > E = F Least

Justify your ranking.

Newton's third law states that A = C, B = D, and E = F since these are action-reaction pairs. The force of gravity depends on the masses and the distances between the masses. The probes' masses are so tiny relative to the mass of the moon that the gravitational forces described in E and F are tiny compared to those in A, B, C, and D. Since the force of gravity also depends on the distance between the objects, Scenario B = D is larger than A = C because the distance between the moon and the probe is smaller in Cases B and D.

NAME ______________________ DATE ____________

Scenario

Angela, Blake, and Carlos are studying the data table to the right, which shows the mass, orbital radius, and orbital period of four planets. They note that the orbital period increases but disagree about why this happens. Their arguments are as follows:

Planet	Mass [10^{24} kg]	Orbital Radius [10^{9} m]	Orbital Period [years]
Mercury	0.330	57.9	0.241
Venus	4.87	108.2	0.616
Earth	5.97	149.6	1.00
Neptune	102	4495.1	164

Angela: "It appears that the more mass a planet has, the longer its period is. This is because more massive objects are more difficult to move, so these objects move slower in their orbits.

Blake: "No, all the planets move at the same speed around the sun, but planets with greater orbital radius must make longer circumference orbits, causing their orbital periods to be greater."

Carlos: "It is the case that farther-radius planets must make farther-circumference orbits, but the farther planets also go slower because there is less gravitational force acting on them."

For this problem, consider one planet of mass m making a circular orbit of radius R around the sun (mass M). Let v represent the speed of the planet as it orbits the sun and T be the orbital period.

Create an Equation

PART A: Beginning with basic equations for gravitational and centripetal force and an equation that relates speed and period of circular motion, derive an expression for the orbital period of this planet in terms of R, M, v, and physical constants as necessary. Note that it may be helpful for Part B for you to number your steps so that they can be referred to later.

<table>
<tr><td>Step 1
$F_g = \frac{GM_P M_S}{R^2}$</td><td>The force of gravity between the sun and one of the planets is:</td></tr>
<tr><td>Step2
$\frac{GM_P M_S}{R^2} = \frac{M_P v^2}{R}$</td><td>It is the gravitational force in this case, which is the centripetal force, holding the planet in orbit.</td></tr>
<tr><td>Step 3
$\frac{GM_S}{R} = v^2$</td><td>Dividing both sides by the mass of the planet. And multiplying both sides by R:</td></tr>
<tr><td>Step 4
$\frac{GM_S}{R} = \frac{4\pi^2 R^2}{T^2}$</td><td>Substituting in the equation $\frac{2\pi R}{T}$ for the velocity of the planet:</td></tr>
<tr><td>Step 5
$T^2 = \frac{4\pi^2 R^3}{GM_S}$</td><td>Multiplying both sides by R (careful here—the R doesn't cancel!)</td></tr>
<tr><td>Step 6
$T = \sqrt{\frac{4\pi^2 R^3}{GM_S}}$</td><td>The period of orbit is related to the radius of orbit by the following equation.</td></tr>
</table>

Argumentation

PART B: Your work in Part A can be used to support or refute the arguments of the three students. For each student, explain which aspects of their reasoning is correct (if any) and incorrect (if any) and cite steps of work from Part A (not your final answer) and explain how that step supports or refutes each aspect.

i. Angela	ii. Blake	iii. Carlos
Angela is wrong. The mass of the planet has no effect on the orbital period. The mass of the planet cancels out in the calculation.	Blake is also incorrect. Step 3 shows that the speed that a planet orbits the sun depends on how far it is from the sun. He is correct in his thinking that in $v = \frac{2\pi R}{T}$, the bigger r is the larger T is, but that assumes an incorrect statement that all the planets have the same speed.	Carlos is correct. There is less gravitational force on the planets that are farther from the sun $\left(F_g = \frac{GM_1M_2}{R^2}\right)$ and there is a slower speed for farther planets according to Step 3.